LEAST-SQUARES FINITE ELEMENT TECHNOLOGY IN FLUID DYNAMICS AND STRUCTURAL MECHANICS

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The use of least-squares principles lead to variationally unconstrained minimization problems possessing many of the attractive properties that are characteristic of classical Ritz formulations [1-4]. This is in contrast to the weak formulations that are, in general, divorced of any such minimization principles. As a result, finite element models of least-squares type always avoid restrictive compatibility conditions and allow for the use of equal interpolation of all dependent variables. In addition, numerical stabilization of first order operators in convection dominated problems is not necessary when least-squares principles are employed.

In this lecture, we will review the basic theory behind least-squares finite element formulations of linear and nonlinear boundary-value and initial- and boundary-value problems arising in the analysis of fluid and solid mechanics problems. Attractive features as well as challenges and limitations of the least-squares variational principles will be discussed. We will describe applications of least-squares principles to linear problems, which always produce symmetric positive-definite coefficient matrices. In the determination of the natural frequencies of structures, we will show that least-squares formulations result in quadratic eigenvalue problems. For time dependent problems, least-squares finite element models can be formulated in terms of space-time coupled or decoupled formulations. In the study of nonlinear problems, we will discuss the manner in which the numerical solution and convergence properties are affected when linearization of the governing equations is performed either before or after minimization of the least-squares functional. Solution techniques for nonlinear problems guaranteeing symmetry of the coefficient matrix that are consistent with the variational formulation will be discussed. We will show that low order finite elements tend to lock when reduced integration techniques are not employed. High-order *hp*-version and k-version finite elements on the other hand offer the prospect of highly accurate numerical results, even when coercivity of a given least-squares functional cannot be established. Numerical solutions obtained using least-squares formulations as applied in the analysis of fluids and structures will be presented. Comparisons will be made directly with results obtained using weak formulations.

References

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